

Global features of proton-neutron interactions and symmetry energy

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Abstract

We study global features of proton-neutron (p - n) interactions and symmetry energy over a wide range of nuclei, using a schematic model interaction with four forces proposed recently. Calculations are performed by the BCS approximation in $N, Z=20$ -50 and $N, Z=50$ -82 regions. The experimental double differences of binding energies and symmetry energy are reproduced quite well. It is shown that the isoscalar p - n interactions with all J are indispensable for explaining the binding energies of not only $N \approx Z$ but also $N > Z$ nuclei in the $A=40$ -160 region.

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In recent years, the study of proton-neutron (p - n) interactions has been a hot topic in the proton rich side of nuclide table. The p - n interactions are expected to become strong in $N \approx Z$ nuclei where valence protons and neutrons in the same orbits have large overlaps of the wavefunctions, and to play very important roles in the structure of these nuclei [1]. The double differences of binding energies, which were investigated for a different purpose before [2–4], have been recognized as useful measures to study the p - n interactions [5–7]. Their observed values display characteristic behaviors near $N = Z$ and reveal properties of the p - n interactions. We have analyzed what p - n interactions cause the characteristic behaviors in a previous paper [7]. One of our conclusions is that the double differences of binding energies trace a curve $40/A$ on the average and the average value is attributed to the isoscalar p - n interaction, almost to the J independent isoscalar p - n force. The investigation, however, is carried out in the $N \approx Z$ nuclei where a large j subshell dominates, i.e., $f_{7/2}$ and $g_{9/2}$ shell nuclei. On the other hand, the curve $40/A$ observed in the double differences of binding energies continues till heavy nuclei. This suggests that considerably strong isoscalar p - n interaction persists over a wide range of nuclei with $N > Z$.

The competition of the isoscalar p - n interaction with the isovector one is observed in the energy spectrum as well as the binding energy [8–17]. It was discussed that the degeneracy of $T = 0$ and $T = 1$ states in odd-odd $N = Z$ nuclei are explained by a near balance of the isovector pairing and symmetry energy [14]. The experimental data indicate that the symmetry energy accompanied by the so-called Wigner energy is proportional to $T(T+1)$ with a common coefficient $a(A)$, i.e., $E_{sym} + E_W = a(A)T(T+1)/A$ where $T = |T_Z| = |N - Z|/2$. It has been shown that the Wigner energy is originated in the isoscalar p - n interaction [18,19] but cannot be solely explained in terms of the $J = 1$ isoscalar p - n interaction [15,19]. These investigations have suggested the importance of isoscalar p - n interactions with various J in the symmetry energy and Wigner energy. This is confirmed in the single j shell model calculation for $f_{7/2}$ shell nuclei in our previous paper [7]. Our model reproduces very well the empirical symmetry energy coefficient $a(A)$. The result indicates that the symmetry energy coefficient is explained by the J independent isoscalar p - n force and $J = 0$ isovector pairing force. In this letter, we extend our discussion to a wide range of nuclei. The empirical formula of Seeger [20,21] or Duflo and Zucker [22] for $a(A)$ has a mass dependence $(1 - \eta A^{-1/3})$. This A dependence combined with the curve $40/A$ of the average double differences of binding energies will give a good guide to determine the A dependence of our isoscalar p - n force strength.

Our previous study [7] has been carried out using a schematic interaction composed of four forces which is extended from the pairing plus QQ force. This schematic interaction reproduces well the low-lying states and binding energies of $N \approx Z$ nuclei where a large j subshell dominates, and also var-

ious nuclear properties, such as the double differences of binding energies, symmetry energy, Wigner energy, odd-even mass difference and two-proton separation energy. The p - n interactions play leading roles in these properties, especially the J independent isoscalar p - n force is essential for the properties which we consider here. The schematic interaction with four force parameters is easy to extend the model space. In this letter, we investigate systematics of the double differences of binding energies and symmetry energy in both of $N \approx Z$ and $N > Z$ nuclei with $N, Z = 20-50$ and $N, Z = 50-80$. We perform this many j shell calculation in the BCS approximation.

The Hamiltonian we proposed in Ref. [17] has an isospin-invariant form as follows:

$$H = H_{\text{sp}} + H_{\text{P}_0} + H_{\text{QQ}} + H_{\text{P}_2} + H_{\pi\nu}^{\tau=0}, \quad (1)$$

$$H_{\text{sp}} = \sum_{\alpha\rho} (\epsilon_a - \lambda) c_{\alpha\rho}^\dagger c_{\alpha\rho}, \quad (2)$$

$$H_{\text{P}_J} = -\frac{1}{2} g_J \sum_{M\kappa} \sum_{a \leq b} \sum_{c \leq d} p_J(ab) p_J(cd) A_{JM1\kappa}^\dagger(ab) A_{JM1\kappa}(cd), \quad (3)$$

$$H_{\text{QQ}} = -\frac{1}{2} \chi \sum_M \sum_{ab\rho} \sum_{cd\rho'} q(ab) q(cd) : B_{2M\rho}^\dagger(ab) B_{2M\rho'}(cd) : \quad (4)$$

$$H_{\pi\nu}^{\tau=0} = -k^0 \sum_{a \leq b} \sum_{JM} A_{JM00}^\dagger(ab) A_{JM00}(ab), \quad (5)$$

with

$$A_{JM\tau\kappa}^\dagger(ab) = \frac{[c_{\alpha\rho}^\dagger c_{\beta\rho'}^\dagger]_{JM}^{\tau\kappa}}{\sqrt{1 + \delta_{ab}}}, \quad B_{JM\rho}^\dagger(ab) = [c_{a\rho}^\dagger c_{b\rho}]_{JM}. \quad (6)$$

Here $p_0(ab) = \sqrt{(2j_a + 1)} \delta_{ab}$ and $p_2(ab) = q(ab)/b^2 = (a \| r^2 Y_2 / b^2 \| b) / \sqrt{5}$ where b^2 is the harmonic-oscillator range parameter. This Hamiltonian has only four parameters, g_0, g_2, χ , and k^0 .

We consider the model space ($1f_{7/2}, 2p_{3/2}, 1f_{5/2}, 2p_{1/2}, 1g_{9/2}$) for $N, Z = 20-50$ region and ($2d_{5/2}, 1g_{7/2}, 3s_{1/2}, 2d_{3/2}, 1h_{11/2}$) for $N, Z = 50-82$ region. The lowest single-particle energy in each region, is determined as $\epsilon_{f_{7/2}} - \lambda = B(^{41}\text{Ca}) - B(^{40}\text{Ca}) = -8.3633$ MeV and $\epsilon_{d_{5/2}} - \lambda = B(^{103}\text{Sn}) - B(^{102}\text{Sn}) = -10.265$ MeV from the experimental binding energies. The other single particle energies are chosen so as to be the same level spacings as those used by Kisslinger and Sorensen [23]. The pairing force strength g_0 and the quadrupole force strength χ are adjusted to fit approximately the experimental odd-even mass differences and the lowest $J = 2^+$ energies in quasiparticle Tamm-Dancoff approximation, respectively,

$$g_0 = 24A^{-1}, \quad \chi = 350A^{-5/3}b^{-4} \quad \text{for } N, Z = 20 - 50 \text{ region}, \quad (7)$$

$$g_0 = 24A^{-1}, \quad \chi = 450A^{-5/3}b^{-4} \quad \text{for } N, Z = 50 - 82 \text{ region}. \quad (8)$$

The quadrupole pairing force strength g_2 can be determined by the same relation $g_2 = 0.2g_0$ as that used by Hara *et al.* [24]. We shall fix the strength k^0 of the isoscalar p - n force (5) in the following discussion.

Let us first analyze the experimental values of the m th double difference of binding energies defined by the following equation [7],

$$\delta V^{(m)}(Z, N) = \delta^{(m)} B(Z, N), \quad (9)$$

where $B(Z, N)$ is the nuclear binding energy and the operator $\delta^{(m)}$ is defined as

$$\begin{aligned} \delta^{(m)} f(Z, N) = & -\frac{1}{m^2} [f(Z, N) - f(Z, N - m) \\ & - f(Z - m, N) + f(Z - m, N - m)]. \end{aligned} \quad (10)$$

Figure 1(a) shows the plot of $\delta V^{(1)}(Z, N)$ as a function of $A = N + Z$ for nuclei in the mass region $A=16$ -210. We can see two separate groups in the figure, namely, one is for the even A nuclei (dots) and the other is for the odd A nuclei (crosses). The patterns of dots and crosses show shell effect at Z or $N=28, 40, 50, 82$ and are symmetric with respect to the average curve $40/A$. The double difference of binding energies $\delta V^{(1)}(Z, N)$ can be approximately written as

$$\delta V^{(1)}(Z, N) = \frac{40}{A} + (-1)^A I', \quad (11)$$

where I' denotes a deviation from the average $40/A$. The large values of $\delta V^{(1)}(Z, N)$ for even- A nuclei (circled dots) at $N = Z$ are striking below $A = 80$. The data of another double difference of binding energies $\delta V^{(2)}(Z, N)$ are plotted in Fig. 1(b) as a function of $A = N + Z$. Our definition of $\delta V^{(2)}$ has the opposite sign to Brenner's one [6]. The $\delta V^{(2)}$ values show a different behavior from the $\delta V^{(1)}$ ones. We see large scatters of dots and crosses for $A < 80$, which show $\delta V^{(2)}$ of $N \approx Z$ nuclei. The values of $\delta V^{(2)}$ at $N = Z$ are especially large as that of $\delta V^{(1)}$. If we neglect the scattered dots and crosses in $N \approx Z$ nuclei, $\delta V^{(2)}$ varies rather smoothly. This smooth trend is clear for $A > 80$ and continues up to heavy nuclei. This is due to the fact that there are no stable $N \approx Z$ nuclei with $A > 80$. The systematic decrease of $\delta V^{(2)}$ with increasing mass A can be traced by the same average curve $40/A$ as that of $\delta V^{(1)}$. The deviations from the curve $40/A$ are small and shell effect disappears for $\delta V^{(2)}$. In Ref. [7], we analyzed $\delta V^{(1)}$ and $\delta V^{(2)}$ for probing the p - n

interactions in $N \approx Z$ nuclei of $g_{9/2}$ and $f_{7/2}$ subshells. The analysis reveals different contributions of isoscalar and isovector p - n interactions to the double differences of binding energies. The average curve $40/A$ of $\delta V^{(1)}$ and $\delta V^{(2)}$ is attributed almost to the isoscalar p - n force $H_{\pi\nu}^{\tau=0}$ introduced, while the p - n part of $P_0 + QQ + P_2$ force produces the deviation from the curve $40/A$ in $\delta V^{(1)}(Z, N)$.

Since $\delta V^{(2)}$ comes from the J independent isoscalar p - n force $H_{\pi\nu}^{\tau=0}$, the average curve $40/A$ of $\delta V^{(2)}$ reflect the A dependence of the force strength k^0 . The interaction energy of $H_{\pi\nu}^{\tau=0}$ in the state with the total valence nucleon number n and isospin T is given by

$$U_{\pi\nu}^{\tau=0} = \langle H_{\pi\nu}^{\tau=0} \rangle = -\frac{1}{2}k^0 \left\{ \frac{n}{2} \left(\frac{n}{2} + 1 \right) - T(T+1) \right\}. \quad (12)$$

Table 1 The coefficient of empirical symmetry energy.

a_{sym} (MeV)	References
28.1	Ring and Schuck [25]
23.3	Droplet model by Myers [26]
25.0	Bohr and Mottelson [27]
20.65 (1.0 - 2.32/ $A^{1/3}$)	Green [28], Seeger [20]
30.59 (1.0 - 1.76/ $A^{1/3}$)	Seeger [21]
33.60 (1.0 - 1.52/ $A^{1/3}$)	J.Duflo and A.P.Zucker [22]

If we calculate the double difference of the interaction energies $U_{\pi\nu}^{\tau=0}$ (which we denote by $\delta^{(2)}U_{\pi\nu}^{\tau=0}$) by substituting $U_{\pi\nu}^{\tau=0}$ for f in Eq. (10), we can expect $\delta^{(2)}U_{\pi\nu}^{\tau=0} \approx \delta V^{(2)}$. We discussed in Ref. [7] that the average curve $40/A$ of $\delta V^{(2)}$ suggests the force strength $k^0 = 80/A$ in the lowest order. The simple form $k^0 = 80/A$, however, cannot well reproduce the curve $40/A$ for $\delta^{(2)}U_{\pi\nu}^{\tau=0}$ as shown in Fig. 2. The disagreement demands a higher order correction. According to our analysis [7], the symmetry energy is also governed by the isoscalar p - n force $H_{\pi\nu}^{\tau=0}$. Let us look at the symmetry energy coefficient $a_{\text{sym}} = a(A)/4$ in order to get a hint on the A dependence of k^0 . Table 1 lists the values of a_{sym} proposed by various authors. The elaborate coefficient in Refs. [20–22,28] has a mass dependence as the form $(1 - \eta/A^{1/3})$ (the lowest A dependence $1/A$ of the symmetry energy is not included in the definition of a_{sym}). The following form of k^0 , therefore, deserves to be tested:

$$k^0 = \frac{\zeta}{A} (1.0 - \eta/A^{1/3}), \quad (13)$$

where ζ and η are parameters. Figure 2 shows the values of $\delta^{(2)}U_{\pi\nu}^{\tau=0}$ calculated with the use of the force strength (13). The calculated $\delta^{(2)}U_{\pi\nu}^{\tau=0}$ values finely reproduce the curve $40/A$ when we choose the parameters $\zeta = 224.0$, $\eta = 2.2$ for the $N, Z = 20 - 50$ region and $\zeta = 224.0$, $\eta = 3.0$ for the $N, Z = 50 - 82$ region.

The four parameters (g_0, χ, g_2, k^0) of our Hamiltonian were fixed by Eqs. (7), (8) and (13). We calculated the binding energies E of $A = 40 - 140$ nuclei within the BCS approximation, i.e., the expectation values of the Hamiltonian with respect to the BCS states for the A systems. (We did not calculate the binding energies near $N = Z$ nuclei and near the beginning and end of each shell region in order to avoid the error of the BCS approximation.) The double differences of binding energies calculated from the approximate ground-state energies E are shown in Figs. 3(a) and 3(b). The calculated values are nicely around the curve $40/A$. Especially, $\delta^{(2)}E$ reproduces quite well the observed variation of $\delta^{(2)}V$ in Fig. 3(b). We can say that our schematic Hamiltonian can qualitatively explain the behavior of the double differences of binding energies also in many- j shell case.

The double differences of binding energies, $\delta^{(1)}V$ and $\delta^{(2)}V$, are considered as good measures of the effective p - n interactions. In the previous paper [7], we investigated from what p - n interactions the characteristic behavior of $\delta^{(1)}V$ and $\delta^{(2)}V$ come, in terms of the single- j shell model for $N \approx Z$ nuclei. Let us develop the analysis in the present many- j shell calculation. We consider the contributions of respective forces to the ground-state energy in even-even, even-odd, odd-even and odd-odd nuclei, respectively,

$$E(e, e) = \langle 0 | H | 0 \rangle = U_0 + U_{\pi\nu}^{\tau=0}, \quad (14)$$

$$E(e, o) = \langle n | H | n \rangle = U_0 + \Delta_n + U_{\pi\nu}^{\tau=0}, \quad (15)$$

$$E(o, e) = \langle p | H | p \rangle = U_0 + \Delta_p + U_{\pi\nu}^{\tau=0}, \quad (16)$$

$$E(o, o) = \langle pn | H | pn \rangle = U_0 + \Delta_p + \Delta_n + \epsilon_{pn} + U_{\pi\nu}^{\tau=0}. \quad (17)$$

In the BCS approximation, the ground states of the four types of nuclei given above are defined by the quasiparticle vacuum, one quasiparticle and p - n two quasiparticle states,

$$|0\rangle, \quad |n\rangle = a_n^\dagger |0\rangle, \quad |p\rangle = a_p^\dagger |0\rangle, \quad |pn\rangle = a_p^\dagger a_n^\dagger |0\rangle, \quad (18)$$

We denote the ground-state energy of the even-even system $|0\rangle$ by U_0 , the neutron (proton) gap by Δ_n (Δ_p) and the interaction energy of the residual force ($QQ + P_2$) by ϵ_{pn} ,

$$U_0 = \langle 0 | H_{sp} + H_{P_0} + H_{QQ} + H_{P_2} | 0 \rangle, \quad (19)$$

$$\Delta_p = \langle \rho | H_{sp} + H_{P_0} + H_{QQ} + H_{P_2} | \rho \rangle - U_0 \quad (|\rho\rangle = |n\rangle \text{ or } |p\rangle), \quad (20)$$

$$\epsilon_{pn} = \langle pn | H_{QQ} + H_{P_2} | pn \rangle. \quad (21)$$

It is easy to calculate $\delta^{(1)}E$ and $\delta^{(2)}E$ from Eqs. (14-17). We have seen in Fig. 2 that the value of $\delta^{(m)}U_{\pi\nu}^{\tau=0}$ yields the curve $40/A$, when we use the parameters (ζ, η) fixed above. We therefore obtain the relation $\delta^{(1)}E(e, e) = \delta^{(1)}E(o, o) = 40/A - \epsilon_{pn}$ for even A nuclei, $\delta^{(1)}E(e, o) = \delta^{(1)}E(o, e) = 40/A + \epsilon_{pn}$ for odd A nuclei and $\delta^{(2)}E = 40/A$ both for even and odd nuclei. The characteristic behaviors of $\delta^{(1)}V$ and $\delta^{(2)}V$ are clearly understood. Their average values $40/A$ are attributed to the J independent isoscalar p - n force also in the present many- j shell calculation. The deviations from the average curve $40/A$ in $\delta^{(1)}V$ are due to the residual p - n interaction between the last proton and neutron in odd-odd nuclei caused by the $QQ + P_2$ force (the QQ force mainly contributes to it). It is notable that the residual p - n interaction ϵ_{pn} is attractive. This and the relation $E(o, o) - E(e, e) = \Delta_p + \Delta_n + \epsilon_{pn}$ obtained from Eqs. (14) and (17) explain the fact that the binding energy difference $B(o, o) - B(e, e)$ is systematically smaller than the sum of gaps $\Delta_p + \Delta_n$ (Bohr and Mottelson [27] suggested an attractive residual p - n interaction about $20/A$ between the last odd proton and neutron in odd-odd nuclei).

Let us now discuss the symmetry energy E_{sym} and the Wigner energy E_W . We calculated E_{sym} and E_W from the ground-state energies in the BCS approximation. Figure 4 shows the calculated and experimental symmetry energy coefficient $a(A)$ in the expression $E_{sym} + E_W = a(A)T(T+1)/A$. (The A dependence of $a(A) = 4a_{sym}$ extracted by Seeger [21] and Duflo et al. [22] are also shown in Fig. 4.) The figure is plotted separately for "diagonal" and "off-diagonal" regions where neutrons and protons are in the same major shell for the former and in the different major shells for the latter. We estimated the Coulomb-energy-corrected binding energies $B^* = B(\text{exp}) + E_{\text{Coul}}(\text{cal})$ following Comay and Jänecke [31] and extracted the symmetry energy coefficients by the treatment of Jänecke and Comay [32]. The agreement between theory and experiment is very well for the diagonal region of $N, Z = 20 - 50$, and is qualitatively good for the off-diagonal region of $N, Z = 20 - 50$ and the diagonal region of $N, Z = 50 - 80$ where the calculated values are a little smaller than the experimental ones. The coefficient $a(A)$ in the off-diagonal region is approximately constant for the heavier nuclei as expected from the simple liquid-drop model. They are larger than those of the diagonal region, as pointed out by Jänecke and Comay [32]. In our calculation, the single-particle energy $E_{sp} = \langle 0 | H_{sp} | 0 \rangle$ influences the symmetry energy coefficient because of a large energy gap between the major shells. We can also see the shell-closure effect on the symmetry energy as well as the double difference of binding energies $\delta V^{(1)}$.

We analyze the symmetry energy in the same way as the single- j shell model calculation [7]. If the single-particle energies are roughly degenerate, i.e., $\epsilon_a \approx$

ϵ , and $H_{QQ} + H_{P_0}$ can be neglected, the Hamiltonian $H_{sp} + H_{P_0} + H_{\pi\nu}^{\tau=0}$ has the SO(5) symmetry. Then, the ground-state energy is approximated in terms of the total valence nucleon number n and the total isospin T as follows:

$$\begin{aligned}\tilde{E} &= \langle H_{sp} + H_{P_0} \rangle + U_{\pi\nu}^{\tau=0} \\ &= (\epsilon - \lambda)n - \frac{1}{2} \left\{ g_0 n \left(\Omega - \frac{n-6}{4} \right) + k^0 \frac{n}{2} \left(\frac{n}{2} + 1 \right) \right\} \\ &\quad + \frac{1}{2} (g_0 + k^0) T(T+1).\end{aligned}\tag{22}$$

The term proportional to $T(T+1)$ in the ground-state energy (22) just corresponds to $E_{sym} + E_W$. The approximate symmetry energy coefficient $\tilde{a}(A)$ is given by

$$\tilde{a}(A) = \frac{1}{2} (g_0 + k^0) A.\tag{23}$$

When we use the parameters $g_0 = 24/A$ and $k^0 = 224(1.0 - 2.2/A^{1/3})/A$ in the *fpg* region, the symmetry energy coefficient in the above SO(5) limit becomes $\tilde{a}_{\text{sym}} = \tilde{a}(A)/4 = 31(1.0 - 1.987/A^{1/3})$ from Eq. (23). This agrees considerably well with the empirical symmetry energy $30.59(1.0 - 1.76/A^{1/3})$ extracted by Seeger [21]. We can, therefore, say that the symmetry energy and Wigner energy are originated nearly in the J independent isoscalar p-n force and the $J = 0$ isovector pairing force. The two contributions to \tilde{a}_{sym} are, for instance, 12.3 MeV and 3.0 MeV respectively in the $A = 60$ system. The isoscalar p - n force $H_{\pi\nu}^{\tau=0}$ plays a significant role in reproducing the experimental symmetry energy and the Wigner energy.

In conclusion, the systematic behaviors of the double differences of binding energies, symmetry energy and Wigner energy are reproduced well by the schematic interaction $P_0 + QQ + P_2 + H_{\pi\nu}^{\tau=0}$ within the BCS approximation, over the wide range of nuclei. It was shown that the J independent isoscalar p-n force is crucial for nuclear properties not only in $N \approx Z$ nuclei but also in $N > Z$ nuclei far from the $N = Z$ line. The analysis also tells that the residual interaction between the last proton and neutron in odd-odd nuclei consists in that of the QQ force. We estimated the effect of the ground-state correlation in the framework of the Random-Phase approximation. The calculations, however, did not change the conclusions mentioned above.

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Figure captions

Fig. 1 The double differences of binding energies derived from the experimental masses in the region $A = 16 - 210$: (a) $\delta V^{(1)}(Z, N)$ as a function of $A = N + Z$; (b) $\delta V^{(2)}(Z, N)$ as a function of $A = N + Z$. The dots stand for even A nuclei, and the crosses for odd A nuclei. The curve $40/A$ is also plotted.

Fig. 2 The calculated values of $\delta^{(2)}U_{\pi\nu}^{\tau=0}(Z, N)$ as a function of $A = N + Z$ in $N, Z=20-50$ and $N, Z=50-82$ regions.

Fig. 3 The calculated values of $\delta^{(1)}E(Z, N)$ and $\delta^{(2)}E(Z, N)$ as a function of $A = N + Z$ in $N, Z=20-50$ and $N, Z=50-82$ regions.

Fig. 4 The symmetry energy coefficient in $N, Z=20-50$ and $N, Z=50-82$ regions. The calculated and experimental values are denoted by the open and filled circles, respectively. The full lines shows the empirical formula by Seeger [21], and the dotted line the empirical formula by Duflo and Zucker [22].







